Name and Surname : ....

Grade/Class : 12/.........   **Mathematics Teacher** : ....

Hudson Park High School

TENTANDO SUPERABIS

GRADE 12

MATHEMATICS

June Paper 2

Marks : ____ / 150

Time : 3 hours

Examiner : SLT

Date : ___/___20__

Moderator(s) : FRD and PHL

INSTRUCTIONS

1. Illegible work, in the opinion of the marker, will earn zero marks.
2. Number your answers clearly and accurately, exactly as they appear on the question paper.
3. **NB** ◦ **Leave *1 line* open between each of your answers.**
4. **NB** **Fill in the details requested on the front of this Question Paper and the Answer Booklet.**
   Hand in your submission in the following manner :
   - **Question Paper (on top)**
   - **Answer Booklet (below)**

   Do not staple your Question Paper and Answer Booklet together.
5. Employ relevant formulae and show all working out. Answers alone may not be awarded full marks.
6. (Non-programmable and non-graphical) Calculators may be used, unless their usage is specifically prohibited.
7. Round off answers to 2 decimal places, where necessary, unless instructed otherwise.
8. If (Euclidean) Geometric statements are made, reasons must be stated appropriately.

QUESTION 1 [8 marks]

1. For the data given below :

17	20	21	25	29	30	35	41	56	60	70	85	88

1.1. Determine the

1.1.1.1. the mean, $\bar{x}$ 1

1.1.1.2. the median, M 1 2

1.1.2. Hence, comment on the distribution of the data. 2 (4)

1.2. Determine the

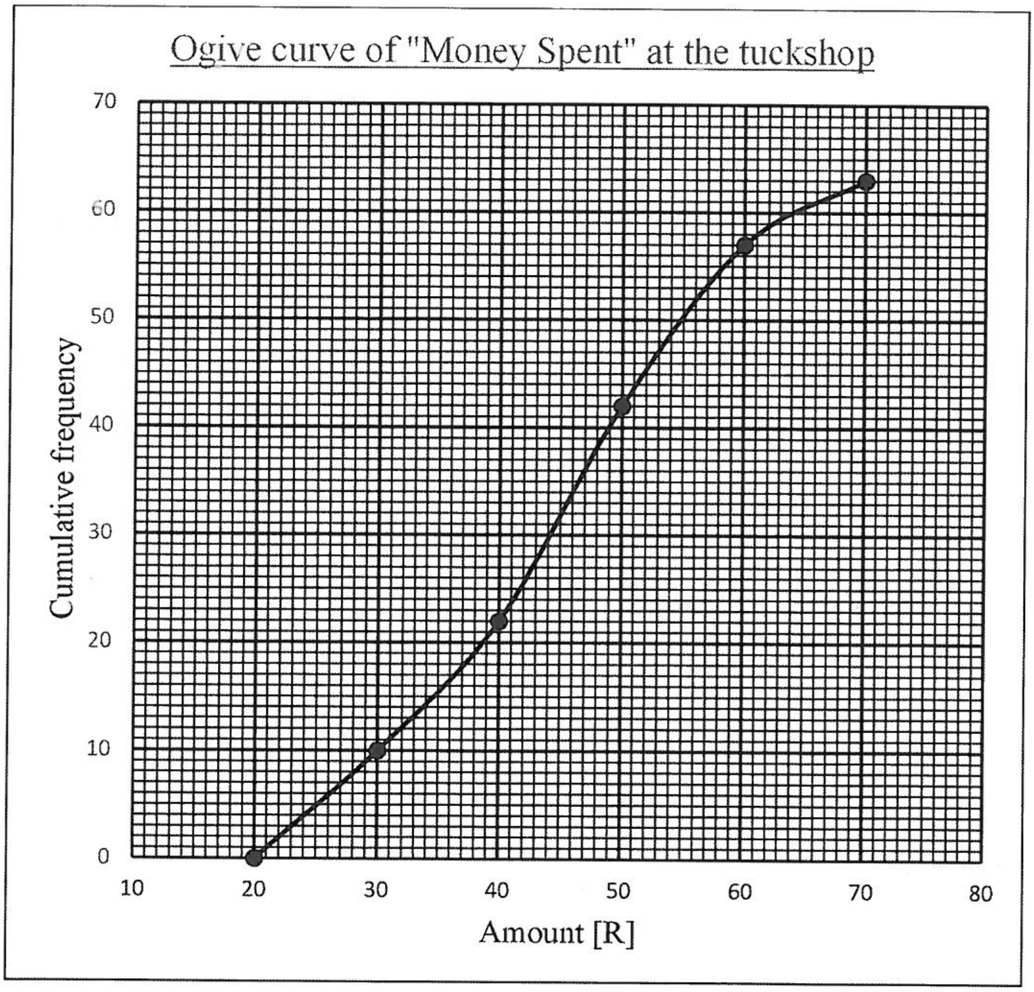
1.2.1. standard deviation, σ 1

1.2.2. percentage of values lying within one standard deviation of the mean. 3 (4)

QUESTION 2 [6 marks]

2. The amount of money, in Rands, that learners spent while visiting the tuckshop of a certain day was analysed to reveal :

Amount [R]	Frequency
$20 < x \leq 30$	a
$30 < x \leq 40$	12
$40 < x \leq 50$	20
$50 < x \leq 60$	b
$60 < x \leq 70$	6



2.1. How many learners visited the tuckshop on the day of the analysis ? (1)

2.2. Determine the values of a and b. (2)

2.3. State the modal class. (1)

2.4. Use the ogive to estimate the number of learners who spent more than R 53 at the tuckshop on the day of the analysis. (2)

QUESTION 3 [6 marks]

3. An oratory competition was held with six speakers participating. Two judges scored each speaker as follows (out of 50) :

Speaker	1	2	3	4	5	6
Judge 1 (x)	45	10	15	20	30	25
Judge 2 (y)	37	15	8	13	35	20

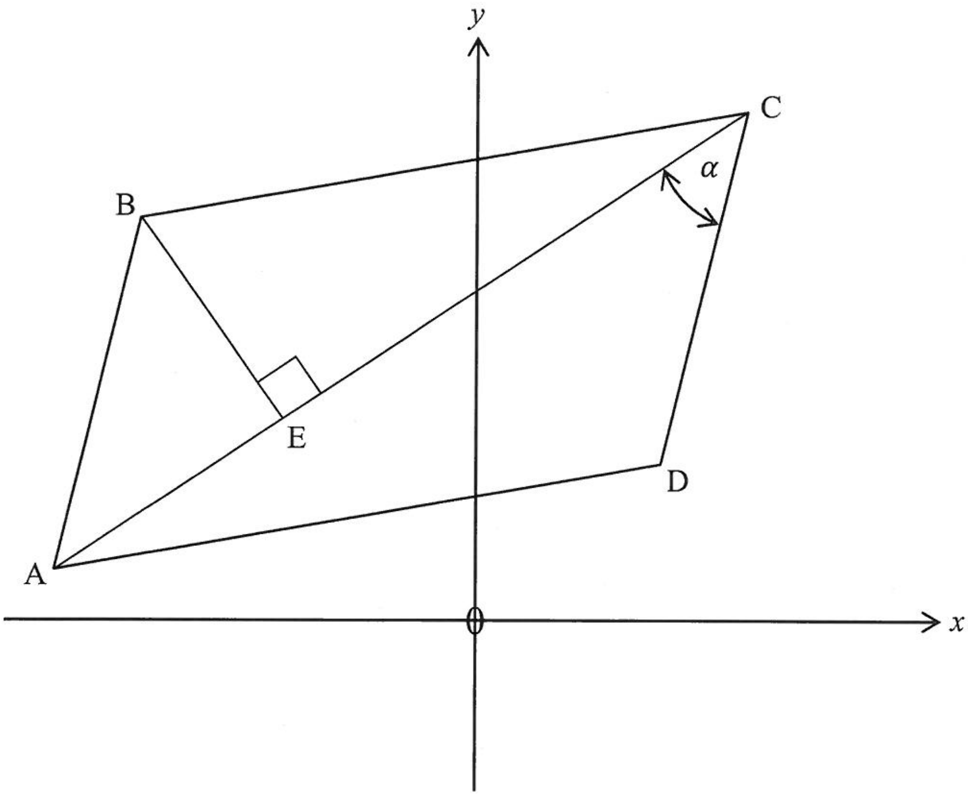
3.1. Determine the equation of the last squares regression line for the scores given by the judges. (3)

3.2. Are the judges consistent in their scoring of the speakers ? Justify your answer with *relevant* statistics. (2)

3.3. A seventh speaker entered late for the oratory competition. Judge 2 scored them as a 30. Estimate what Judge 1 would have scored them as, to the nearest whole number. (1)

QUESTION 4 [22 marks]

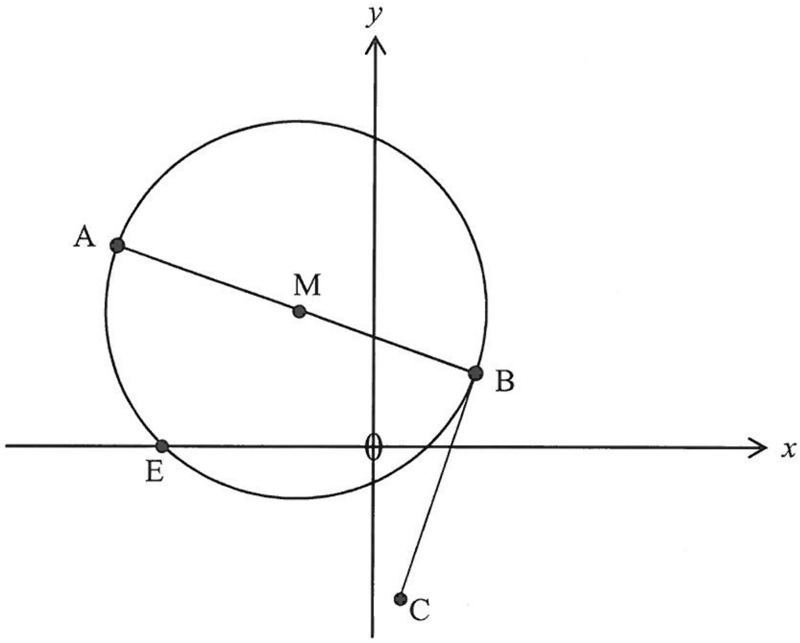
4. ABCD is a parallelogram with AC ⊥ BE. A(−6; 1), B(−5; 6), C(3; 7) and $A\hat{C}D = \alpha$.



4.1.	Determine the equations of		
4.1.1	AC	3	
4.1.2.	BE	3	(6)
4.2.	Calculate the coordinates of E, showing that they will be E(−3; 3).		(3)
4.3.	Calculate the lengths (in surd form if necessary) of :		
4.3.1.	AC	2	
4.3.2.	BE	1	(3)
4.4.	Determine the area of parallelogram ABCD.		(3)
4.5.	Calculate the magnitude of α.		(5)
4.6.	Write down the coordinates of D.		(2)

QUESTION 5 [16 marks]

5. M is the centre of the circle whose equation is $(x+1)^2+(y-6)^2=45$.
B$(5;3)$ and C$(1;-5)$.



5.1. Determine the coordinates of

5.1.1. M 1

5.1.2. A 2

5.1.3. E 3 (6)

5.2. Is BC a tangent to the circle at point B ? Justify your answer with all the relevant calculations and reasons. (5)

5.3. If D$\left(d\,;\,-7\frac{5}{6}\right)$, B and C are collinear, calculate the value of d. (3)

5.4. If the circle was moved 2 units vertically upwards and the radius was doubled, what would its new equation be ? (2)

QUESTION 6 [17 marks]

6.1. Given : $\cos(A - B) = \cos A \cos B + \sin A \sin B$

Use the given formula to derive the formula for $\sin(A - B)$. (3)

6.2. Given : $p \tan 26° - 1 = 0$, determine the following without the use of a calculator :

6.2.1. $\sin 86°$ 5

6.2.2. $\sin 13°$ 3

6.2.3. $\tan 2096°$ 3 (11)

6.3. If $\cos 2x = \frac{3}{5}$, determine $\cos(-x)$ without the use of a calculator. (3)

QUESTION 7 [16 marks]

7. Given $f(x) = -\sin 2x$ and $g(x) = \cos(x + 60°)$.

7.1. On the given set of axes, sketch rough graphs of f and g, for $x \in [-180°; 180°]$. (6)

7.2. For f, write down the

7.2.1. range 1

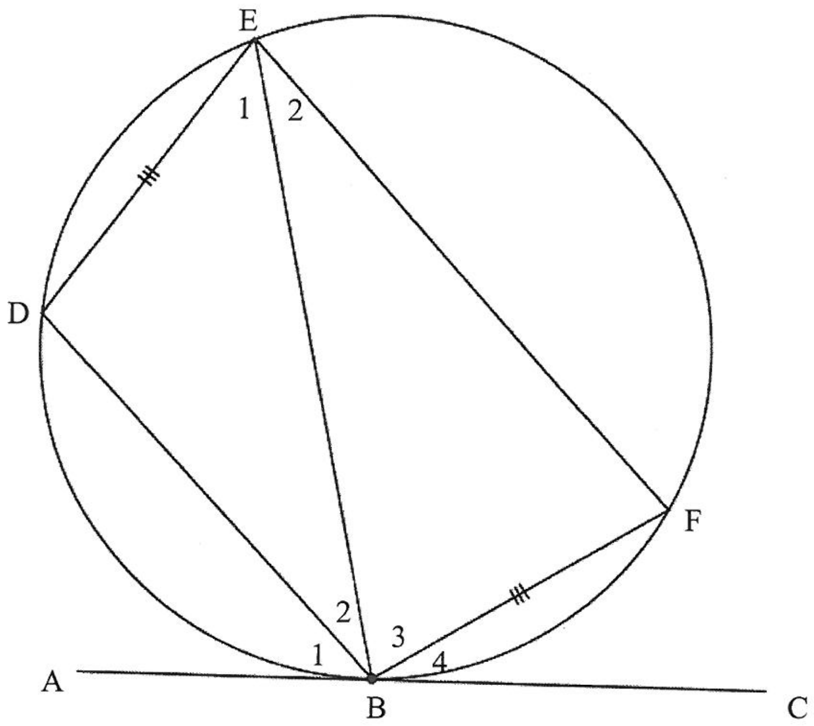
7.2.2. period 1 (2)

7.3.1. Calculate the general solution of $f(x) = g(x)$. 5

7.3.2. Now, solve for x if $f(x) > g(x)$ and $x \in [-180°; 180°]$. 3 (8)

QUESTION 8 [11 marks]

8. DE = FB = 5 units, EF = 6 units, BE = 7 units, ABC is a tangent to the circle at point B and $\hat{D} > 90°$.



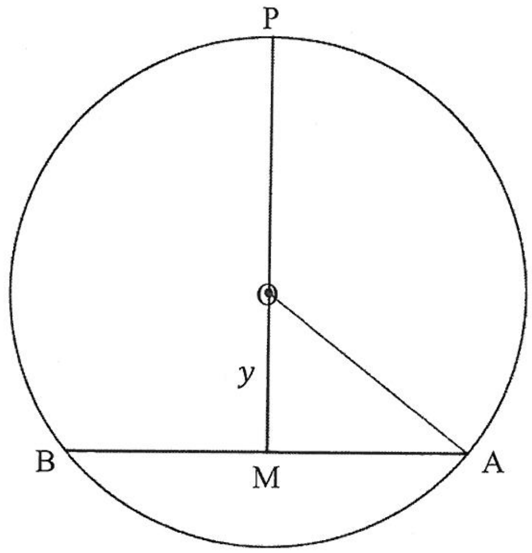
Calculate

8.1. $\hat{E}_2$ (3)

8.2. $\hat{B}_1$ (8)

QUESTION 9 [6 marks]

9. M is the midpoint of chord AB. OM = y , AB = 20 units and $\frac{\text{PM}}{\text{OM}} = \frac{5}{2}$.



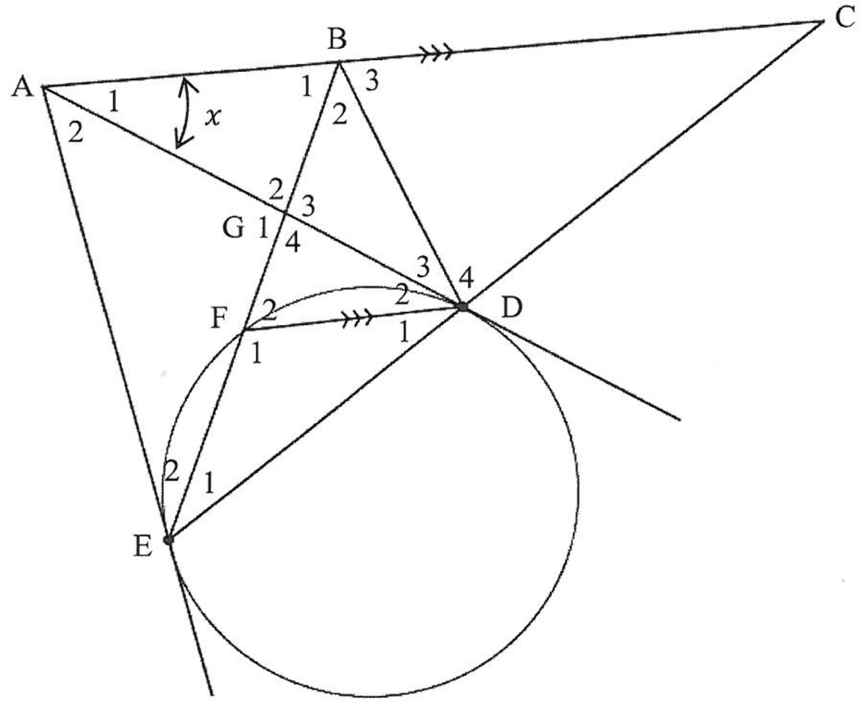
9.1. Write down the length of MA. (1)

9.2. Give the reason why OM $\perp$ AB. (1)

9.3. Calculate the value of y. (4)

QUESTION 10 [9 marks]

10. AE and AD are tangents to the circle at points E and D respectively. ABC // FD. $\widehat{A}_1 = x$.



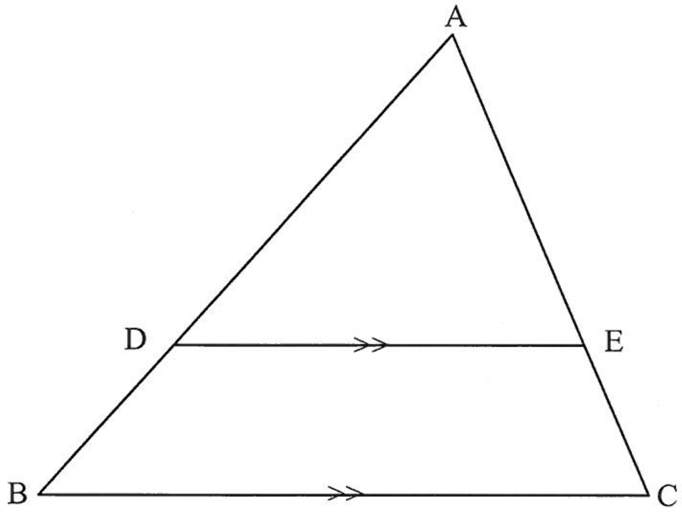
Prove that :

10.1. ABDE is a cyclic quadrilateral. (4)

10.2. If it is further given that EF = FD, that AE = CD. (5)

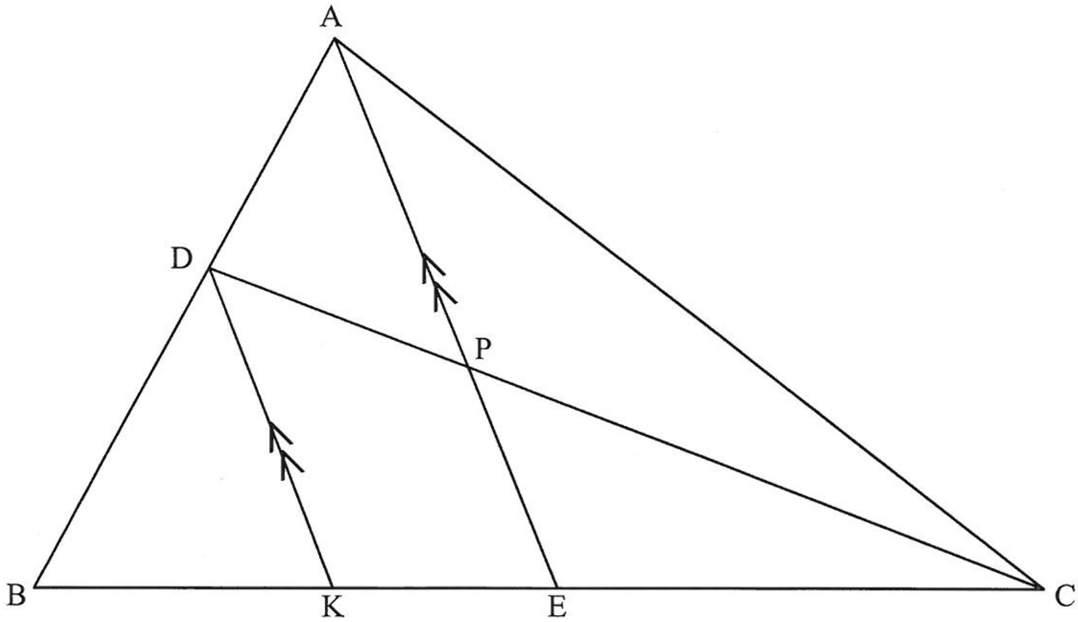
QUESTION 11 [11 marks]

11.1. In the diagram, DE // BC.



Prove the theorem which states that : $\frac{AD}{DB} = \frac{AE}{EC}$ (6)

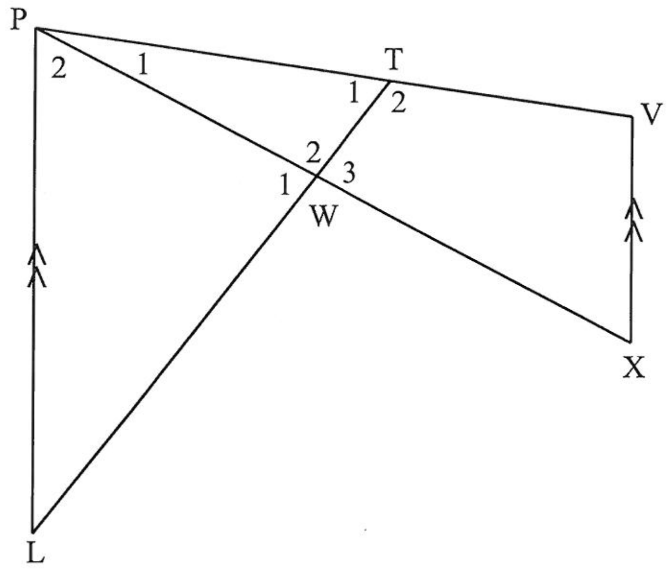
11.2. $\frac{AD}{BD} = \frac{2}{3}$ and $\frac{BC}{EC} = \frac{7}{3}$. KD // EPA.



Calculate : $\frac{CP}{PD}$ (5)

QUESTION 12 [8 marks]

12. PT = 6 units, TV = 4 units, VX = 4 units, XW = 7 units, WP = 5 units, TW = 2 units and PL // VX.



Prove that :

12.1. ΔPTW /// ΔPXV (4)

12.2. PL is a tangent to the circle passing through points P, W and T. (4)

5. INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$